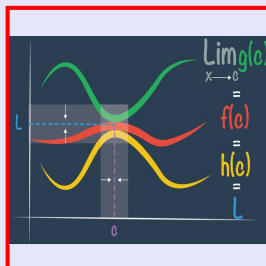


# Calculus I

## Lecture 27



Feb 19-8:47 AM

Class Quiz 12

Find  $\frac{dy}{dx}$ 

Do not simplify

1)  $y = \sqrt{1+x^2}$

$y = (1+x^2)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$$

2)  $y = x \cdot \sin\left[\frac{1}{x}\right]$

$$\frac{dy}{dx} = 1 \cdot \sin\frac{1}{x} + x \cdot \cos\frac{1}{x} \cdot \frac{-1}{x^2} = \sin\frac{1}{x} - \frac{1}{x} \cos\frac{1}{x}$$

Oct 15-7:15 AM

Given  $\frac{1}{x} + \frac{1}{y} = 1$

1) Find  $y$   $\frac{1}{y} = 1 - \frac{1}{x} \rightarrow \frac{y}{1} = \frac{x}{x-1}$   
 $\frac{1}{y} = \frac{x-1}{x} \rightarrow y = \frac{x}{x-1}$

2) Find  $\frac{dy}{dx}$   $\frac{dy}{dx} = \frac{1(x-1) - x \cdot 1}{(x-1)^2}$

Another Method  $\frac{dy}{dx} = \frac{-1}{(x-1)^2}$  ✓

$\frac{1}{x} + \frac{1}{y} = 1$   
 $x^{-1} + y^{-1} = 1$

Implicit Diff.  
 $-1x^{-2} - 1y^{-2} \cdot \frac{dy}{dx} = 0$   
 $\frac{dy}{dx} = \frac{-x^{-2}}{-y^{-2}} = -\frac{y^2}{x^2}$

$\frac{dy}{dx} = -\frac{(\frac{x}{x-1})^2}{x^2} = -\frac{x^2}{(x-1)^2 x^2} = -\frac{1}{(x-1)^2}$

Oct 15-7:39 AM

$2\sqrt{x} + \sqrt{y} = 3$

Find  $\frac{dy}{dx}$

$2x^{1/2} + y^{1/2} = 3$

$2 \cdot \frac{1}{2} x^{-1/2} + \frac{1}{2} y^{-1/2} \cdot \frac{dy}{dx} = 0$

$x^{-1/2} + \frac{1}{2} y^{-1/2} \frac{dy}{dx} = 0$

$2x^{-1/2} + y^{-1/2} \frac{dy}{dx} = 0$

$y^{-1/2} \frac{dy}{dx} = -2x^{-1/2}$

$\frac{dy}{dx} = -\frac{2x^{-1/2}}{y^{-1/2}}$

$\frac{dy}{dx} = -\frac{2y^{1/2}}{x^{1/2}}$

$\frac{dy}{dx} = -\frac{2\sqrt{y}}{\sqrt{x}}$

$\frac{dy}{dx} = -2 \cdot \sqrt{\frac{y}{x}}$

Oct 15-7:46 AM

Find  $\frac{dy}{dx}$  if  $y^5 + x^2y^3 = 1 + x^4y$

$$\frac{d}{dx}[y^5] + \frac{d}{dx}[x^2y^3] = \frac{d}{dx}[1] + \frac{d}{dx}[x^4y]$$

$$5y^4 \frac{dy}{dx} + \frac{d}{dx}[x^2] \cdot y^3 + x^2 \cdot \frac{d}{dx}[y^3] = 0 + \frac{d}{dx}[x^4] \cdot y + x^4 \frac{d}{dx}[y]$$

$$5y^4 \frac{dy}{dx} + 2xy^3 + x^2 \cdot 3y^2 \frac{dy}{dx} = 4x^3y + x^4 \frac{dy}{dx}$$

$$[5y^4 + 3x^2y^2 - x^4] \frac{dy}{dx} = 4x^3y - 2xy^3$$

$$\frac{dy}{dx} = \frac{4x^3y - 2xy^3}{5y^4 + 3x^2y^2 - x^4}$$

Oct 15-7:55 AM

Find  $\frac{dy}{dx}$ :

$$\tan \frac{x}{y} = x + y$$

$$\sec^2 \frac{x}{y} \cdot \frac{1 \cdot y - x \cdot \frac{dy}{dx}}{y^2} = 1 + \frac{dy}{dx}$$

$$\text{LCD} = y^2$$

$$\sec^2 \frac{x}{y} \cdot (y - x \frac{dy}{dx}) = y^2 + y^2 \frac{dy}{dx}$$

$$y \sec^2 \frac{x}{y} - x \sec^2 \frac{x}{y} \frac{dy}{dx} = y^2 + y^2 \frac{dy}{dx}$$

$$y \sec^2 \frac{x}{y} - y^2 = [y^2 + x \sec^2 \frac{x}{y}] \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{y \sec^2 \frac{x}{y} - y^2}{y^2 + x \sec^2 \frac{x}{y}}$$

Oct 15-8:04 AM

Find eqn of the tan. line to the graph  
 $x^2 + xy + y^2 = 3$  at  $(1, 1)$   
 Tilted Ellipse

$m = \frac{dy}{dx} \Big|_{(1,1)}$

$$\frac{d}{dx}[x^2] + \frac{d}{dx}[xy] + \frac{d}{dx}[y^2] = \frac{d}{dx}[3]$$

Product Rule

$$2x \left[ +1 \cdot y + x \cdot \frac{dy}{dx} \right] + 2y \cdot \frac{dy}{dx} = 0$$

at  $(1, 1)$

$$2(1) + 1 + 1 \cdot m + 2 \cdot m = 0$$

$$3 + 3m = 0 \quad m = -1$$

$$y - 1 = -1(x - 1)$$

$$\boxed{y = -x + 2}$$

Oct 15-8:11 AM

Find eqn of the tan. line to the graph  
 of  $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$  at  $(0, \frac{1}{2})$   
 Cardioid

$m = \frac{dy}{dx} \Big|_{(0, \frac{1}{2})}$

$$2x + 2y \cdot \frac{dy}{dx} = 2(2x^2 + 2y^2 - x)$$

$$[4x + 4y \frac{dy}{dx} - 1]$$

$$2x + 2y \frac{dy}{dx} = 2(2x^2 + 2y^2 - x) [4x + 4y \frac{dy}{dx} - 1]$$

$$2(0) + 2(\frac{1}{2}) m = 2(2 \cdot 0^2 + 2(\frac{1}{2})^2 - 0) [4 \cdot 0 + 4 \cdot \frac{1}{2} m - 1]$$

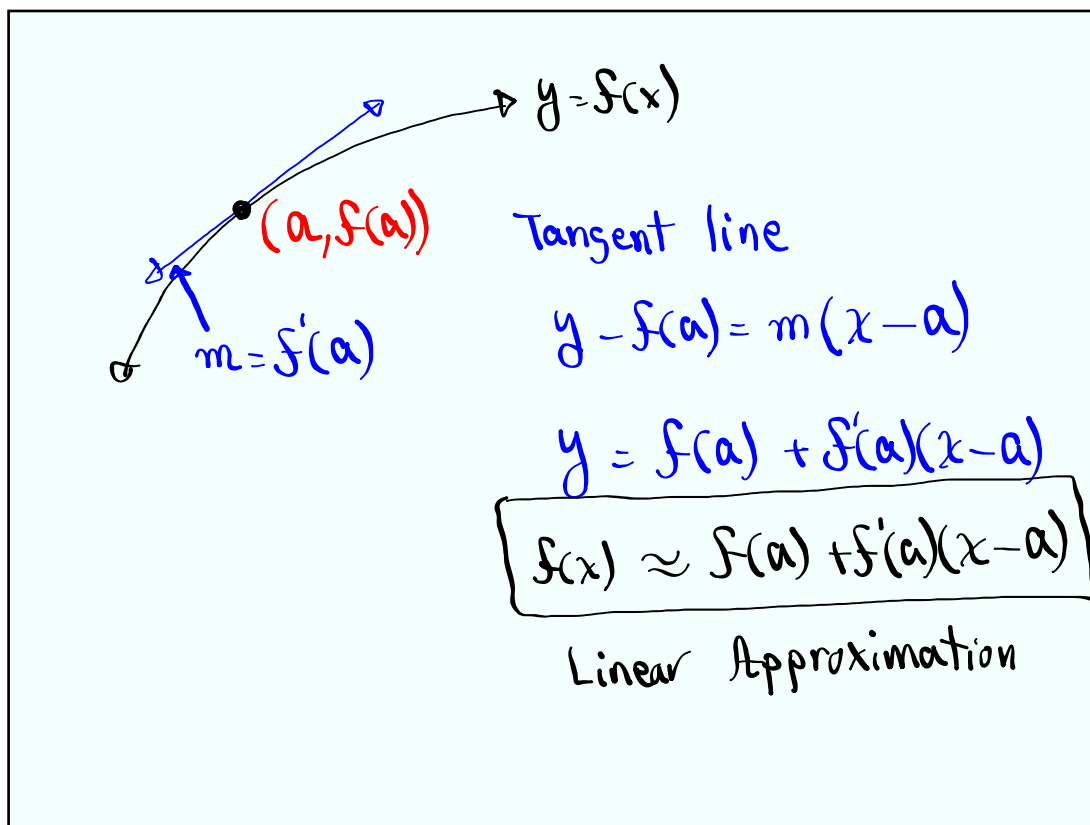
$$m = 1 [2m - 1]$$

$$m = 2m - 1 \quad \rightarrow \quad 2m - m = 1$$

$$\boxed{m = 1}$$

$$y - \frac{1}{2} = 1(x - 0) \quad \boxed{y = x + \frac{1}{2}}$$

Oct 15-8:17 AM



Oct 15-8:27 AM

Estimate  $\sqrt{4.1}$

Your calc  
 $\sqrt{4.1} \approx \underline{2.024845673}$

$f(x) = \sqrt{x}$   
 $a = 4$   
 $f'(x) = \frac{1}{2\sqrt{x}}$

$f(x) \approx f(a) + f'(a)(x - a)$   
 $\sqrt{x} \approx \sqrt{4} + \frac{1}{2\sqrt{4}}(x - 4)$   
 $\sqrt{x} \approx 2 + \frac{1}{4}(x - 4)$

$\sqrt{4.1} \approx 2 + \frac{1}{4}(4.1 - 4)$   
 $= 2 + \frac{1}{4}(.1)$   
 $= 2 + \frac{1}{40} = \frac{81}{40}$   
 $= 2.025$

Oct 15-8:30 AM