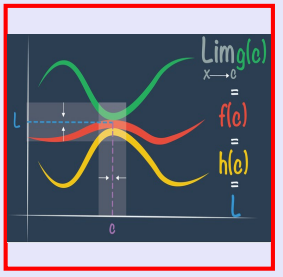
Calculus I

Lecture 27



Feb 19-8:47 AM

Class Quiz 12

Find $\frac{dy}{dx}$ Do not Simplify

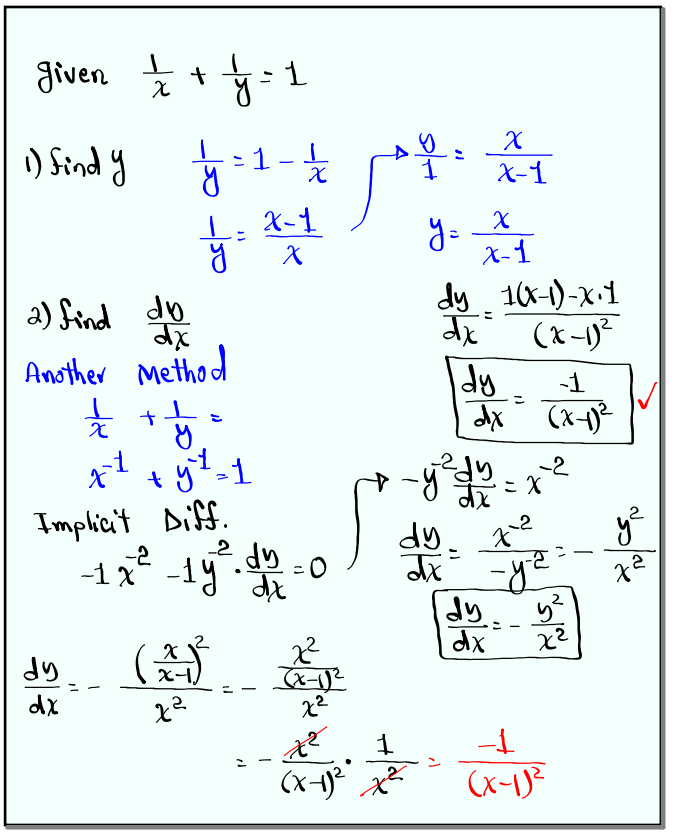
1) $y = \sqrt{1+x^2}$ $\quad y = (1+x^2)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$$

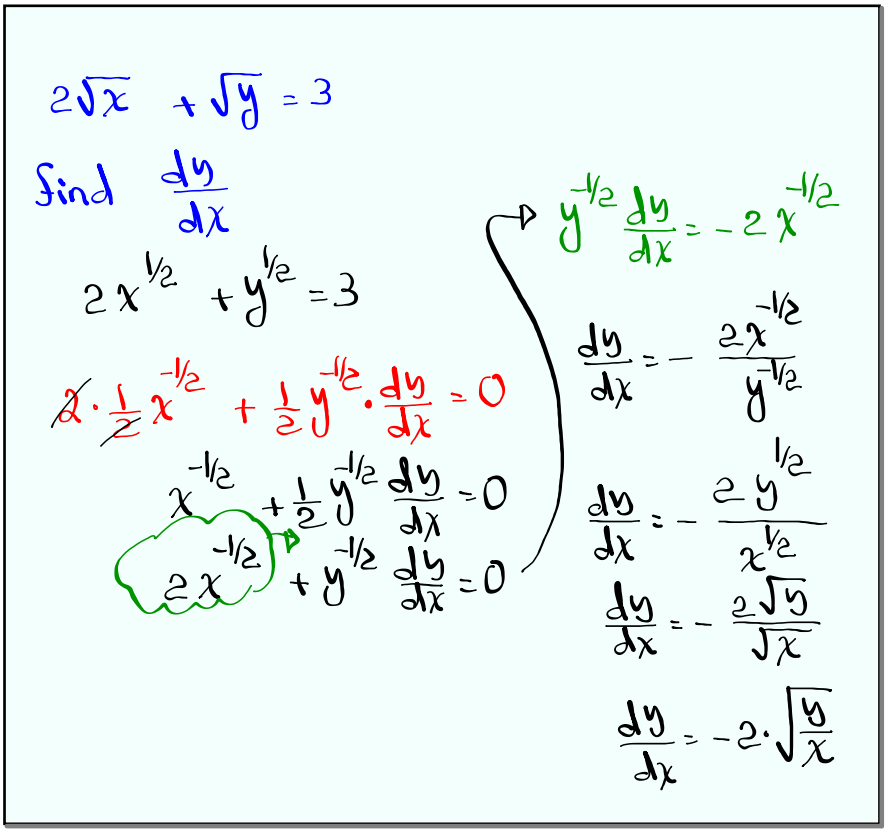
2) $y = x \cdot \sin\frac{1}{x}$

$$\frac{dy}{dx} = 1 \cdot \sin\frac{1}{x} + x \cdot \cos\frac{1}{x} \cdot \frac{-1}{x^2} = \sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}$$

Oct 15-7:15 AM



Oct 15-7:39 AM



Oct 15-7:46 AM

Find $\frac{dy}{dx}$ if $y^5 + x^2y^3 = 1 + x^4y$

$\frac{d}{dx}[y^5] + \frac{d}{dx}[x^2y^3] = \frac{d}{dx}[1] + \frac{d}{dx}[x^4y]$

$5y^4\frac{dy}{dx} + \frac{d}{dx}[x^2]\cdot y^3 + x^2\cdot\frac{d}{dx}[y^3] = 0 + \frac{d}{dx}[x^4]\cdot y + x^4\cdot\frac{d}{dx}[y]$

$5y^4\frac{dy}{dx} + 2xy^3 + x^2\cdot 3y^2\cdot\frac{dy}{dx} = 4x^3y + x^4\frac{dy}{dx}$

$\left[5y^4 + 3x^2y^2 - x^4\right]\frac{dy}{dx} = 4x^3y - 2xy^3$

$$\frac{dy}{dx} = \frac{4x^3y - 2xy^3}{5y^4 + 3x^2y^2 - x^4}$$

Oct 15-7:55 AM

Find $\frac{dy}{dx}$:

$\tan\frac{x}{y} = x + y$

$\sec^2\frac{x}{y}\cdot\frac{1\cdot y - x\cdot\frac{dy}{dx}}{y^2} = 1 + \frac{dy}{dx}$

LCD $= y^2$

$\sec^2\frac{x}{y}\cdot\left(y - x\frac{dy}{dx}\right) = y^2 + y^2\frac{dy}{dx}$

$y\sec^2\frac{x}{y} - x\sec^2\frac{x}{y}\frac{dy}{dx} = y^2 + y^2\frac{dy}{dx}$

$y\sec^2\frac{x}{y} - y^2 = \left[y^2 + x\sec^2\frac{x}{y}\right]\frac{dy}{dx}$

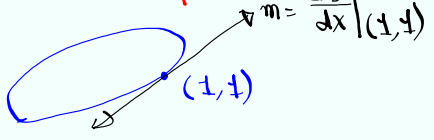
$$\frac{dy}{dx} = \frac{y\sec^2\frac{x}{y} - y^2}{y^2 + x\sec^2\frac{x}{y}}$$

Oct 15-8:04 AM

Find eqn of the tan. line to the graph

$x^2 + xy + y^2 = 3$ at $(1,1)$

Tilted Ellipse



$\frac{d}{dx}[x^2] + \frac{d}{dx}[xy] + \frac{d}{dx}[y^2] = \frac{d}{dx}[3]$

Product Rule

$2x + \left[1 \cdot y + x \cdot \frac{dy}{dx}\right] + 2y \cdot \frac{dy}{dx} = 0$

at $(1,1)$

$2(1) + 1 + 1 \cdot m + 2 \cdot m = 0$

$3 + 3m = 0 \quad m = -1$

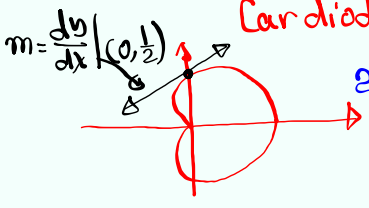
$y - 1 = -1(x-1)$

$\boxed{y = -x + 2}$

Oct 15-8:11 AM

Find eqn of the tan. line to the graph of $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at $(0, \frac{1}{2})$

Cardiod



$2x + 2y \cdot \frac{dy}{dx} = 2(2x^2 + 2y^2 - x) \cdot \left[4x + 4y\frac{dy}{dx} - 1\right]$

$2x + 2y\frac{dy}{dx} = 2(2x^2 + 2y^2 - x)\left[4x + 4y\frac{dy}{dx} - 1\right]$

$2(0) + 2(\frac{1}{2})m = 2(2 \cdot 0^2 + 2(\frac{1}{2})^2 - 0)\left[4 \cdot 0 + 4 \cdot \frac{1}{2} \cdot m - 1\right]$

$m = 1[2m - 1]$

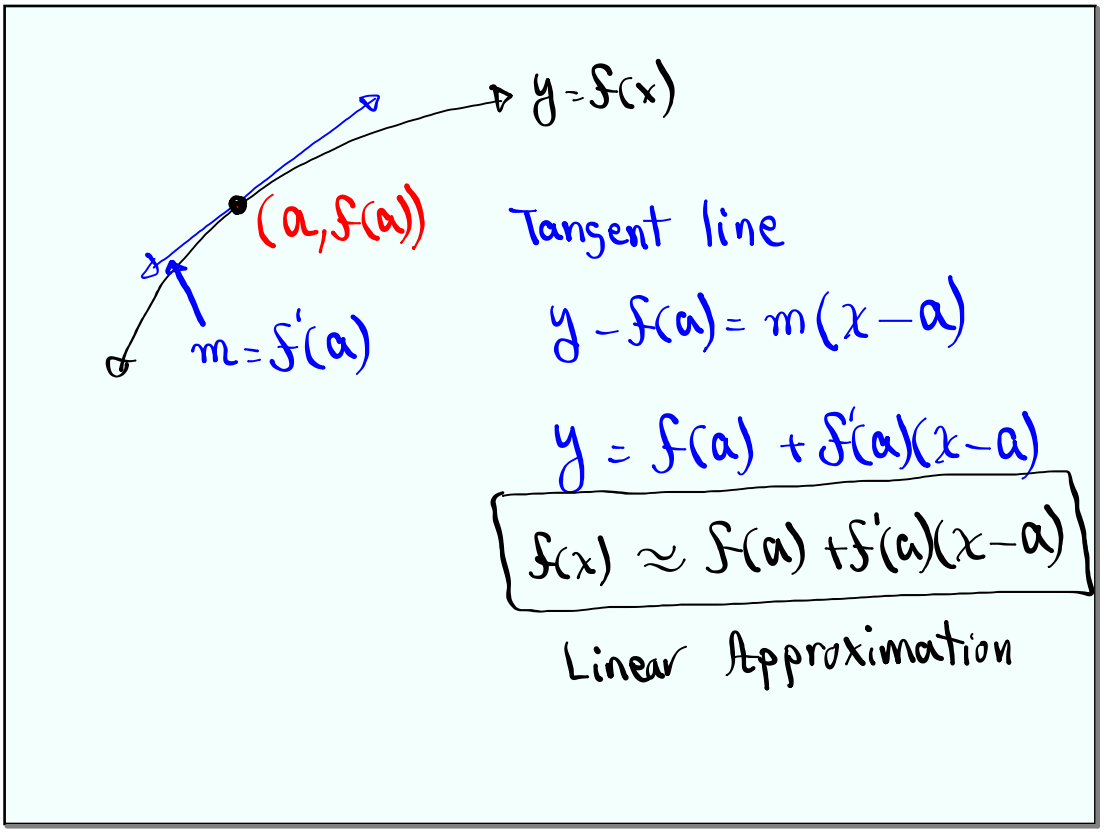
$m = 2m - 1$

$2m - m = 1$

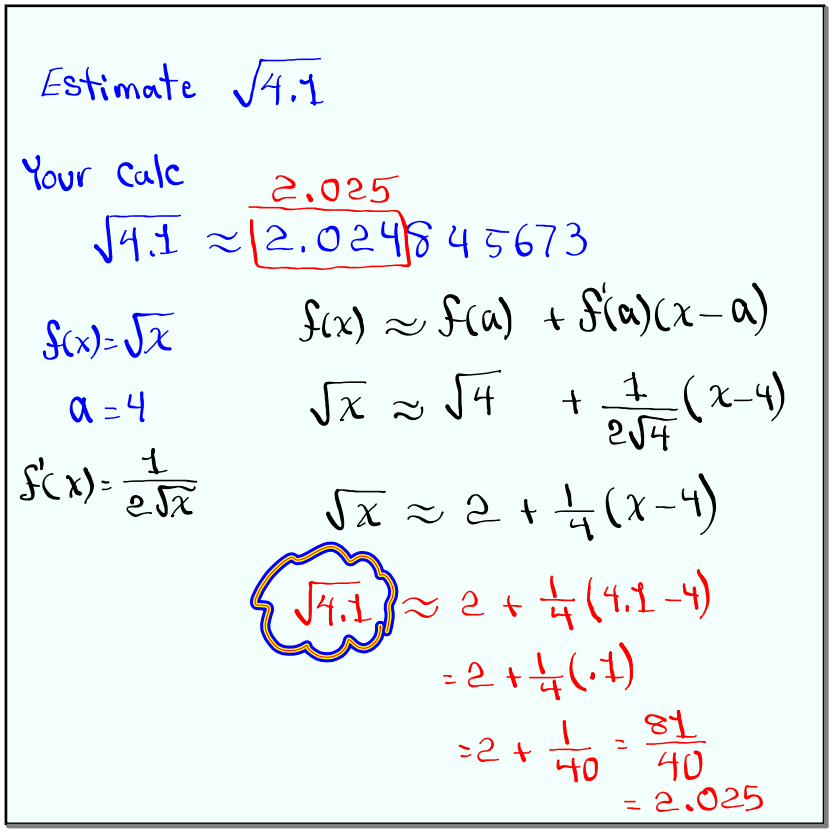
$\boxed{m = 1}$

$y - \frac{1}{2} = 1(x - 0) \quad \boxed{y = x + \frac{1}{2}}$

Oct 15-8:17 AM



Oct 15-8:27 AM



Oct 15-8:30 AM